SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Show that the function $u = \sin x \cosh y + 2\cos x \sinh y$ satisfies Laplace's equation and find the corresponding analytic function $u + iv$.
17. State and prove the Abel's limit theorem.
18. Let $f(z)$ be analytic function within and on the boundary C of a simply connected region D and let z_0 be any point within C. Then prove that
$$F'(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^2}dz.$$
19. State and prove the Taylor's theorem.
20. State and prove the Rouche's theorem.

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G.T.N. ARTS COLLEGE (AUTONOMOUS)

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END SEMESTER EXAMINATION – APRIL 2020

Programme : M.Sc. Mathematics **Date : 16.09.2020**

Course Code: 17PMAC41 **Time : 10.00 am to 1.00 pm.**

Course Title : Complex Analysis **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. A curve F_g given by $z(t) = x(t) + iy(t)$, $\alpha \le t \le \beta$ is called a Jordan arc if $z(t)$ is ______________.

   [a] one-one [b] onto

   [c]into [d] many-one

2. Two families of curves are said to form an ________ system if they intersect at right angles at each of their points of intersections.

   [a] orthonormal [b] orthogonal

   [c] intersect [d] perpendicular

3. The radius of convergence of $\sum(-1)^n (z-2i)^n / n$ is __________.

   [a] 0 [b] -1

   [c] 1 [d] ∞

4. The sum function $f(z)$ of the power series $\sum a_n z^n$ represents an analytic function inside the ________ of convergence.

   [a] radius [b] circumference

   [c] region [d] circle

5. If $f(z)$ is analytic at all points within and on the closed contour C then $\int_C f(z)dz =$ __________.

[a] 0 [b] 1
[c] -1 [d] ∞

6. The parametric equation of the circle with center *'a'* and radius *r* is $|z-a|$ _________ r .

[a] < [b] =
[c] > [d] $\neq$

7. A function which has poles as its only singularities in the finite part of the plane is said to be a _________ function.

[a] entire [b] analytic
[c] meromorphic [d] removable

8. The ____________ of a function is a point at which the function ceases to be analytic.

[a] residue [b] pole
[c] removable [d] singularity

9. The pole of $\frac{z^2}{z^2+a^2}$ is z = ___________.

[a] ia [b] -ia
[c] $\pm ia$ [d] a

10. The value of $\frac{1}{2\pi i}\int_{|z|=3}\frac{e^z}{z-2}dz$ is _________.

[a] 1 [b] e^2
[c] -1 [d] ∞

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Verify whether the real and imaginary parts of w = sinz satisfy Cauchy-Riemann equations.

[OR]

b) Show that the function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function.

12. a) Find the region of convergence of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{z^{2n-1}}{(2n-1)!}$.

[OR]

b) State and prove the Cauchy's criterion for uniform convergence.

13. a) Evaluate by Cauchy's integral formula $\int_C \frac{dz}{z(z+\pi i)}$ where c is $|z+3i|=1$.

[OR]

b) State and prove the Cauchy's inequality.

14. a) State and prove the uniqueness theorem.

[OR]

b) Find the Laurent's series of the function $f(z)=\frac{1}{(z^2-4)(z+1)}$ valid in the region $1<|z|<2$.

15. a) State and prove the Cauchy's residue theorem.

[OR]

b) Find the residues of the function $\frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane.

15. a) If T is normal, then prove that the $M_i^{'}$s are pairwise orthogonal.

[OR]

b) If T is normal, then prove that each M_i reduces T.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. State and Prove Hahn – Banach Theorem.
17. State and prove the open mapping theorem.
18. Let H be a Hilbert space and let f be an arbitrary functional in H^*. Then prove that there exists a unique vector y in H such that f(x) = <x, y>.
19. Prove that the adjoint operation $T \rightarrow T^*$ on B(H) has the following properties:
    a) $(T_1 + T_2)^* = T_1^* + T_2^*$
    b) $(\alpha T)^* = \bar{\alpha}\, T^*$
    c) $(T_1 T_2)^* = T_2^* \, T_1^*$
    d) $T^{**} = T$
    e) $\|T^*\| = \|T\|$
    f) $\|T^*T\| = \|T\|^2$
20. Prove that two matrices in A_n are similar iff they are the matrices of a single operator on H relative to different bases.

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END SEMESTER EXAMINATION - APRIL 2020

Programme : M.Sc. Mathematics **Date : 19.09.2020**

Course Code: 17PMAC42 **Time : 10.00 a.m to 1.00 p.m**

Course Title : Functional Analysis **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. The set of bounded linear operator on a normed linear space X with a suitable norm, a ____________.
   [a] Metric [b] Normed linear space
   [c] Banach Space [d] Linear Space
2. Every complete subspace of a normed linear space is _____________.
   [a] Compact [b] Closed
   [c] Bounded [d] Continuous
3. If B is a reflexive Banach space then its closed unit sphere S is ______.
   [a] compact [b] connected
   [c] complete [d] weakly compact
4. Let B and $B^{'}$ be two banach spaces and if T is continuous linear transformation of B and $B^{'}$, then T is _______________ mapping.
   [a] Open [b] Closed
   [c] One-One [d] Onto

5. If unitary operators on H form a __________.

[a] Subgroup [b] Monoid

[c] Cyclic [d] Group

6. If S is a non-empty subset of a Hilbert space then $S^{\perp}$ =_________.

[a] S [b] $S^{\perp\perp}$

[c] $S^{\perp\perp\perp}$ [d] - $S^{\perp}$

7. The orthogonal complement of subspace of a Hilbert space is ________.

[a] Continuous [b] Connected

[c] Compact [d] Complete

8. If N is a normal operator on H then $\|N^2\|$ = _________.

[a] $\|N\|^2$ [b] - $\|N\|^2$

[c] $2^{\|N\|}$ [d] - $\|N^2\|$

9. If X is an eigen vector of T, then any non-zero vector x in H such that Tx = λx is called an eigen vector corresponding to the eigen value ______.

[a] 1 [b] λ

[c] 0 [d] ∞

10. An operator T on H is normal ⇔ its adjoint T^* is a _______ in T.

[a] Dimension [b] Basis

[c] Span [d] Polynomial

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Let M be a linear subspace of a real normed linear space N, and let f be a functional defined on M. If x_0 is a vector not in M, and if $M_0 = M+[x_0]$ is the linear subspace spanned by M and x_0, then prove that f can be extended to a functional f_0 defined on M_0 such that $\|f_0\| = \|f\|$.

[OR]

b) Prove that if N is an normed linear space and x_0 is a non-zero vector in N, then there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.

12. a) If P is a projection on a Banach space B, and if M and N are its range and null space, prove that M and N are closed linear subspaces of B such that $B = M \oplus N$.

[OR]

b) State and Prove The Closed Graph Theorem.

13. a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

[OR]

b) If M and N are closed linear subspaces of Hilbert space H such that $M \perp N$, then prove that the linear subspace M + N is also closed.

14. a) If A_1 and A_2 are self-adjoint operators on H, then prove that their product A_1A_2 is self-adjoint ⇔ $A_1A_2 = A_2A_1$.

[OR]

b) If N is a normal operator on H, then prove that $\|N^2\| = \|N\|^2$.

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END SEMESTER EXAMINATION – APRIL 2020

Programme :M. Sc. Mathematics **Date : 17.09.2020**

Course Code : 17PMAC42 **Time : 10:00 am to 1.00 pm.**

Course Title : Functional Analysis **Max Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. Let X be a normed linear space and let x, y$\in$X. Then $\left| \|x\| - \|y\| \right| \leq$______.

[a] $\|x+y\|$ [b] $\|y+2x\|$

[c] $\|x-y\|$ [d] $|x-y|$

2. Let N be a non-zero normed linear space then N is a Banach space iff

__________.

[a] $\{x \mid \|x\| < 1\}$ is complete [b] $\{x \mid \|x\| = 1\}$ is complete

[c] $\{x \mid \|x\| > 1\}$ is complete [d] $\{x^2 \mid \|x\| = 1\}$ is complete

3. Let N be a normed linear space. Then find out the correct statement.

[a] the conjugate space N^* is also normed linear space.

[b] N^{**} the second conjugate of N

[c] $(N^*)^*$ is the conjugate space of N^*

[d] all the above are true

4. If B and B' are the Banach spaces and if T is a continuous linear transformation of B onto B', then T is ________.

[a] closed [b] bounded

[c] closed map [d] open mapping

5. In a Hilbert space H, <x, y> = ________.

[a] $\overline{<x,y>}$ [b] $<y,x>$

[c] $-<x,y>$ [d] $-<y,x>$

6. Two vectors x and y in a Hilbert space H are said to be orthogonal if ________.

[a] $<x,y>\neq 0$ [b] $<x,y>=0$

[c] $|x+y|=0$ [d] $\|x-y\|=0$

7. $\|T^*T\|=$ ______.

[a] $\|T\|^2$ [b] T^2

[c] $-T^2$ [d] 2T

8. If A_1 and A_2 are self- adjoint operators on H, then their product $A_1 A_2$ is self-adjoint iff ________.

[a] $A_1A_2 = -A_2A_1$ [b] $A_1A_2 = -A_1A_2$

[c] $A_1A_2 = A_2A_1$ [d] $A_1A_2 = 0$

9. The dimension of B(H) is ________.

[a] n [b] n^2

[c] n^3 [d] 2n

10. If T is normal then ___________.

[a] the M_i's are pairwise orthogonal [b] each M_i reduces T

[c] the M_i's span H [d] all of the above true

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Let M be a closed linear subspace of a normal linear space N. If the norm of a coset x+M in the quotient space N/M is defined by $\|x+M\| = \inf\{\|x+m\| / m \in M\}$, then prove that N/M is a Banach space if N is a Banach space.

[OR]

b) If N is a normed linear space and x_0 is non-zero vector in N then prove that there exist a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\|=1$.

12. a) If N is a normed linear space then prove that the closed unit sphere S^* in N^* is a compact Hausdroff space in the weak * topology.

[OR]

b) State and prove the closed graph theorem.

13. a) State and prove the Schwarz inequality.

[OR]

b) If M is a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$.

14. a) Prove that in the adjoint operation $T \to T^*$ on B(H) has the following properties:

i) $(T_1+T_2)^* = T_1^* + T_2^*$ ii) $(\alpha T)^* = \bar{\alpha} T^*$

iii) $(T_1T_2)^* = T_2^* T_1^*$ iv) $T^{**} = T$

[OR]

b) If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other then prove that $N_1 + N_2$ and N_1N_2 are normal.

15. a) If T is normal then prove that M_i's are pairwise orthogonal.

[OR]

b) If T is normal then prove that the M_i's span H.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Let N and N' be normed linear spaces and T a linear transformation of N into N'. Then prove that the following are equivalent to one another.

a) T is continuous

b) T is continuous at the origin in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$.

c) There exists a real number k >0 with the property that

$\|T(x)\| \leq k\|x\| \ \forall x \in N$.

d) if $S = \{x / \|x\| \leq 1\}$ is the closed unit sphere in N_1 then its image T(s) is a bounded set in N'.

17. State and prove the open mapping theorem.

18. Let H be a Hilbert space and let $\{e_i\}$ be an orthonormal set in H then prove the following are equivalent

a) $\{e_i\}$ is complete b) $x \perp \{e_i\} \Rightarrow x = 0$

c) if x is an arbitrary vector in H, then $x = \Sigma < x, \rho_i > \rho_i$

d) if x is an arbitrary vector in H, then $\|x\|^2 = \Sigma |< x, \rho_i|^2$

19. i) If T is an operator on H for which $< Tx, x > = 0 \ \forall x$, then prove that T = 0.

ii) An operator T on H is self-adjoint iff $< Tx, x >$ is real for all x.

20. Prove that two matrices in A_n are similar iff they are the matrices of a single operator on H relative to different bases.

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END SEMESTER EXAMINATION – APRIL 2020

Programme : M. Sc. Mathematics **Date : 18.09.2020**

Course Code: 17PMAE41 **Time : 10:00 am to 1:00 pm.**

Course Title : Mathematical Statistics **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. Let Q(A) be equal to the number of points (x, y) in A. If $Q(A_1) = 16$, $Q(A_2) = 7$ and $Q(A_1 \cup A_2) = 21$, then $Q(A_1 \cap A_2) =$ __________.

   [a] 1 [b] 2

   [c] 3 [d] 0

2. Let $f(x) = \frac{1}{x^2}$, $0 < x < \infty$, 0 elsewhere be the probability density function of X. If $A_1 = \{x : 1 < x < 2\}$. Then $P(A_1) =$ __________.

   [a] 1 [b] ½

   [c] 2 [d] 0

3. If A_1 & A_2 are subsets of A, the conditional probability of the event A_2, given the event A_1 is $P\left(\frac{A_2}{A_1}\right)=$ ________.

[a] $P(A_1 \cup A_2)$ [b] $\frac{P(A_1 \cap A_2)}{P(A_2)}$

[c] $\frac{P(A_1 \cap A_2)}{P(A_1)}$ [d] $P(A_2)$

4. If X and Y are independent random variables, then $\rho=$ ________.

[a] 0 [b] 1

[c] -1 [d] 2

5. In which distribution, the mean and variance are equal?

[a] Binomial [b] Poisson

[c] Normal [d] Gamma

6. The gamma distribution transforms to an exponential distribution with $\alpha=$ ________.

[a] 1 [b] 0

[c] 2 [d] -1

7. In a 't' distribution, the value of $\beta_2=$ ________.

[a] 0 [b] 3

[c] 1 [d] 2

8. Let X have the uniform distribution over the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Then Y = tan X has a ____________ distribution.

[a] Chi square [b] Gamma

[c] Cauchy [d] Normal

9. If r = ½ , then the variance of chi-square distribution is ________.

[a] ½ [b] 2

[c] 1 [d] 0

10. If $\frac{Y_n}{n}$ converges stochastically to 'p', then $1-\frac{Y_n}{n}$ converges stochastically to ________.

[a] p [b] 1-p

[c] 0 [d] 1

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Let the probability density function of X and Y be $f(x,y)=2,\ 0<x<y,\ 0<y<1,\ 0$ elsewhere. Prove or disprove $E(X).E(Y)=E(XY)$.

[OR]

b) State and prove Chebyshev's inequality.

12. a) State and prove Baye's formula for conditional probability.

[OR]

b) Let the joint probability density function of X_1 and X_2 be

$$f(x_1,x_2)=\frac{x_1+x_2}{21},\ x_1=1,2,3;\ x_2=1,2\ \&\ 0\ \textit{elsewhere}.$$

Find the marginal density functions.

13. a) Derive recurrence relation for the moments of the binomial distribution.

[OR]

b) In a chi-square distribution, if $(1-2t)^{-6},\ t<\frac{1}{2}$ is the moment generating function of the random variable X, find $P(X<5.23)$.

14. a) Let X_1, X_2, X_3 be a random sample of size 3 from a distribution that is n(6,4). Determine the probability that the largest sample item exceeds 8.

[OR]

b) Let T have a 't' distribution with 14 degrees of freedom. Determine 'b' so that, $P(-b<T<b)=0.90$.

15. a) Let $\overline{X_n}$ denotes the mean of a random sample of size n from a distribution that has mean μ and positive variance σ^2. Show that $\overline{X_n}$ converges stochastically to μ if σ^2 is finite.

[OR]

b) Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution that is $\chi^2(50)$. Compute an approximate value of $P(49<\overline{X}<51)$.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Find the mean and variance of the distribution that has the distribution function:

$$f(x)=\begin{cases}0 & x<0\\ \dfrac{x}{8}, & 0\le x<2\\ \dfrac{x^2}{16}, & 2\le x<4\\ 1 & 4\le x\end{cases}$$

17. If X and Y have the joint p.d.f. $f(x,y)=\begin{cases}x+y, & 0<x<1,\ 0<y<1\\ 0, & elsewhere\end{cases}$.

Show that the correlation coefficient of X and Y is $\rho=\frac{-1}{11}$.

18. Compute the measures of skewness and kurtosis of a gamma distribution with parameters α & β.

19. Derive student's 't' distribution.

20. State and prove Central Limit theorem.